Homework 4. This homework is listed as problem and then solution.

**Given the following matrix and direction (0,1,2) and (-1,0,2), find another conjugate direction.**

**| 2 3 1 |**

**H= | 4 0 2 |**

**| 1 1 1 |**

Let the conjugate direction be (d1, d2, d3)

Then we must satisfy

| 2 3 1 | | d1 |

(0,1,2) | 4 0 2 | | d2 | =0

| 1 1 1 | | d3 |

And

| 2 3 1 | | d1 |

(-1,0,2) | 4 0 2 | | d2 | =0

| 1 1 1 | | d3 |

So

| d1 |

(8,4,2) | d2 | =0 and

| d3 |

| d1 |

(0,1,1) | d2 | =0

| d3 |

So 8d1+4d2+2 d3=0

d2+d3=0

Let d3=1, so d2=-1 and d1=1/4

So a conjugate direction is (1/4, -1, 2)

**Perform 1 iteration of DFP**

**f(x1,x2) = x12-4x1 +2x22 starting at (1,0) with D1=|2 1|.**

|1 2|

The gradient of f is | 2x1-4 |

| 4x2 |

At (1,0) this is (-2,0)T

d1 = - |2 1| | -2 | = |4 |

| 1 2| | 0 | | -2 |

Solve min (1+4λ)2 - 4(1+4λ)+ (-2λ)2

Taking the derivative results in

2 (1+4λ)4 – 16 -4λ = 0

28λ=8

λ = 2/7

y2 =(15/7,-4/7)

with a gradient of

(2/7,-16/7)

So

p1= |8/7 |

| -4/7 |

q1= |16/7 |

| -4/7 |

D2 = | 2 1 | |8/7| |8/7,-4/7 | | 2 1 | |16/7| |16/7,-4/7 | | 2 1 |

|1 2 | + |-4/7| + |1 2 | |-4/7| | 1 2 |

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|8/7,-4/7| |16/7| |16/7,-4/7 | | 2 1 | |16/7 |

|-4/7 | | 1 2 | |-4/7 |

D2 = | 2 1 | |64/49 -32/49| |28/7| |28/7,8/7 |

|1 2 | + |-32/49 16/49| + |8/7|

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144/49 416/49

D2 = | 2 1 | |4/9 -2/9| |784/49 224/49 |

|1 2 | + |-2/9 1/9| + | 224/49 64/ 49|

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416/49

D2 = |22/9 7/9 | |1.88 .538 |

|7/9 19/9 | + | .538 .154|

D2 = |4.32 1.316 |

|1.316 2.265|

The next direction is

- |4.32 1.316 | | 2/7 | = |-1.77 |

|1.316 2.265| |-16/7| | - 4.8 |

**3. Assume that we have done a conjugate gradient method for a while on x3-y2+xy. The last direction searched was (2,1) and the current point is (3,-1). Perform the next iteration. Also assume that you want to take α = ½ in all situations. Furthermore, assume the largest step size that is allowed is 2, λ≤2.**

The gradient is

|3x2+y |

|-2y+x |

The direction is

-| 26 | |2 | |-25|

-|5 | + ½ | 1 | = |-4.5|

Now minimize

|3 | |-25| in the equation **x3-y2+xy.**

|-1 | + λ |-4.5|

So minimize (3-25λ)3-(-1-4.5λ)2+ (3-25λ)(-1-4.5λ)

Using a single dimension search technique, I got the maximum step size λ=2. So the next point is (-47, -10). We are clearly heading to an unbounded solution.

**4. Do three iterations of the penalty method with a quadratic penalty. Assume that the u’s start at 10 and increase by a multiple of 10 each time.**

**Minimize x2+y2**

**s.t. x+y= 8**

The penalty function takes the form Minimize x2+y2 +u(x+y-8)2

**The first iteration solves**

Minimize x2+y2 +10(x+y-8)2

Taking the gradient and setting equal to 0, we get

2x+ 20(x+y-8)=0

2y+ 20(x+y-8)=0

22x+20y = 160

20x+22y = 160

x= 160/42 and y= 160/42. This clearly is not feasible.

**The second iteration solves**

Minimize x2+y2 +100(x+y-8)2

Taking the gradient and setting equal to 0, we get

2x+ 200(x+y-8)=0

2y+ 200(x+y-8)=0

202x+200y = 1600

200x+202y = 1600

x= 1600/402 and y= 1600/402. This clearly is not feasible.

**The third iteration solves**

Minimize x2+y2 +1000(x+y-8)2

Taking the gradient and setting equal to 0, we get

2x+ 2000(x+y-8)=0

2y+ 2000(x+y-8)=0

2002x+2000y = 1600

2000x+2002y = 1600

x= 1600/4002 and y= 1600/4002. This clearly is not feasible.

However, the optimal solution is 4,4 and so we are clearly converging.

**5. Do three iterations of the barrier method with a 1/g(x) barrier. Assume that the u’s start at 10 and that the multiplicative factor is 1/2.**

**Minimize x2+y2**

**s.t. x+y≥ 8**

So now the barrier function takes the form Minimize x2+y2 +u (1/(x+y-8))

**The first iteration solves**

Minimize x2+y2 +10 (1/(x+y-8))

Taking the gradient and setting equal to 0, we get

2x-10(1/(x+y-8)2)=0

2y-10(1/(x+y-8)2)=0

So

2x=10(1/(x+y-8)2)

And

2y=10(1/(x+y-8)2)

Thus, x=y

So

2x=10(1/(2x-8)2)

2x(2x-8)2=10

8x3-64x2+128x -10=0

x=4.5255 and so y = 4.5255.

Note, I tried to solve this with goal seek, but it gave me .081, which is not in the interior. So I used my own heuristic bisection method to get a solution in the interior.

The error term to check for terminating

The error is 5\*(-1/(4.5255+4.5255-8)) is over 10

So we continue

**The second iteration solves**

Minimize x2+y2 +5 (1/(x+y-8))

Taking the gradient and setting equal to 0, we get

2x-5(1/(x+y-8)2)=0

2y-5(1/(x+y-8)2)=0

So

2x=5(1/(x+y-8)2)

And

2y=5(1/(x+y-8)2)

Thus, x=y

So

2x=5(1/(2x-8)2)

2x(2x-8)2=5

8x3-64x2+128x -5=0

x=4.378 and so y = 4.378.

The error is 5\*(-1/(4.378+4.379-8)) =6.6

**The third iteration solves**

Minimize x2+y2 +2.5 (1/(x+y-8))

Taking the gradient and setting equal to 0, we get

2x-2.5(1/(x+y-8)2)=0

2y-2.5(1/(x+y-8)2)=0

So

2x=2.5(1/(x+y-8)2)

And

2y=2.5(1/(x+y-8)2)

Thus, x=y

So

2x=2.5(1/(2x-8)2)

2x(2x-8)2=2.5

8x3-64x2+128x -2.5=0

x=4.27 and so y = 4.27.

The error is 5\*(-1/(4.27+4.27-8)) =4.6

22x+20y = 160

20x+22y = 160

x= 160/42 and y= 160/42. This clearly is not feasible.